PERGAMON

International Journal of Solids and Structures 36 (1999) 4743–4766

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Seismic soil-structure interaction analysis by direct boundary element methods

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Received 16 June 1997; accepted 11 May 1998

Abstract

Upon establishing the mathematical framework necessary for a proper understanding of the analytical theory, a regularized form of the conventional direct boundary integral equation formulation for three-dimensional elastodynamics is presented for a general anisotropic medium. Founded on the basis of a full decomposition of the Green's functions into regular and singular parts, the alternative boundary integral equation format is compact and without demanding mathematical and numerical complexities such as Cauchy principal values. Extended to deal with general seismic soil-structure interaction problems in semi-infinite media, the formulation is implemented computationally together with a rigorous treatment of singular dynamic multi-layered viscoelastic half-space Green's functions and interfacial boundary tractions arising in typical soil-structure-foundation configurations. A set of new benchmark numerical results are included. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

The general subject of dynamic soil-structure interaction is relevant to a wide range of applications such as earthquake-resistant design of structures, foundations, tunnels and lifelines. One common challenge to these problems is the need to deal with the complex stress wave propagation in a semi-infinite soil or geological medium. In the past decades, significant analytical advances have been made in the subject area in terms of mathematical and computational solutions to the problem. On the mathematical side, one can refer to the work of Luco and Westmann (1972) and Pak and Gobert (1991) as examples on how the classical techniques of singular and dual integral equations can be applied to some fundamental foundation vibration problems. For more general geometries and configurations, use can also be made of the finite element method by virtue of the

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PII: S0020-7683(98)00263-7

developments such as those in Lysmer and Kuhlemeyer (1969) and Tassoulas and Kausel (1983). Stemming from its intrinsic suitability in dealing with unbounded media, however, the boundary element method has gained the recognition in recent years as the logical tool for this class of problems, serving either as an independent candidate (Dominquez and Abascal, 1987; Tassoulas, 1989) or a vital component in hybrid procedures (Bielak et al., 1991; Belytschko and Lu, 1994). Within the boundary element framework (Beskos, 1997), one can generally separate the formulations into two categories: the direct and indirect methods. Direct methods employ the displacement and traction at the boundary as the primary variables (e.g. Rizzo et al., 1985; Karabalis and Beskos, 1987) while indirect methods involve formulating the problem in terms of some auxiliary source distributions which may be fictitious or physical in nature (e.g. Mita and Luco, 1989; Pak and Ji, 1994). Common to both types of formulation is the task of dealing with the singularity and discontinuities of the Green's functions whose accurate evaluation is of utmost importance. To establish the boundary element matrices, such singular solutions must also be integrated adequately over all elemental boundary surfaces. Although numerical implementations in the form of special transformation-quadrature techniques (e.g. Lachat and Watson, 1976; Lean and Wexler, 1985) or ad-hoc off-boundary collocation schemes (e.g. Mita and Luco, 1989) have been proposed to deal with such difficulties, their performance are intimately related to the specific nature of the singularities and discontinuities of the relevant Green's functions and can vary from problem to problem. To pursue a truly rigorous solution for common soil-structure configurations and geometries, one must also be ready to tackle the possibility of singular contact stresses and interfacial tractions which often arise in such mixed boundary value problems in mechanics (Pak and Abedzadeh, 1996). In this exposition, an overview of some fundamental theoretical and computational developments critical to a rigorous application of direct boundary integral equation methods to general seismic soil-structure interaction analysis is presented. Most of the issues and resolutions are, however, equally relevant to indirect boundary element formulations.

The paper begins with a basic examination of the theoretical foundation of the direct boundary integral equation formulation in three-dimensional elastodynamics. Included are some mathematical details which are central to a proper understanding of the method, but yet commonly omitted in past treatments. Upon establishing the conventional boundary integral equation format in terms of Cauchy principal values of surface integrals, an alternative form of the direct method is explored which involves weakly singular integrals only. Specialized to employ half-space Green's functions, the regularized formulation is extended to both wave radiation and scattering problems. To further enhance its engineering applications, a consistent singularity treatment of viscoelastic multi-layered half-space Green's functions and a set of singular edge- and corner-boundary elements to handle unbounded contact tractions at soil-structure interfaces have also been developed. As illustrations, selected numerical results for some benchmark problems are included for general reference and comparisons.

2. Fundamental integral representation

With reference to a Cartesian frame $\{0; \xi_1, \xi_2, \xi_3\}$ for an open regular region Ω, the governing differential field equations in linearized elastodynamics are

$$\tau_{ij,j}+f_i = \rho\ddot{u}_i, \quad \xi\in\Omega \tag{1}$$

from the balance of linear momentum, and

$$\tau_{ij}(\xi, t) = C_{ijkl}(\xi)u_{k,l}(\xi, t) \tag{2}$$

as the stress–strain relationship where C_{ijkl} is the fourth-order elasticity tensor with major and minor symmetries characterizing a general anisotropic medium. In eqns (1) and (2), $u_i(\xi, t)$ is the displacement field, $\tau_{ij}(\xi, t)$ is the Cauchy stress tensor, f_i is the body-force field per unit volume, ρ is the mass density, and ξ and t are, respectively, the spatial and temporal coordinates. The general displacement- and traction-boundary conditions for $\Gamma = \Gamma_u \bigcup \Gamma_t$, the closed boundary of Ω, (see Fig. 1) can be expressed as

$$\begin{aligned} u_i(\xi, t) &= \tilde{u}_i(\xi, \mathrm{t}), \quad \xi\in\Gamma_u, \quad t > 0, \\ t_i(\xi, t; \boldsymbol{n}) &= \tau_{ij}(\xi, t)n_j = \tilde{t}_i(\xi, t), \quad \xi\in\Gamma_t, \quad t > 0, \end{aligned} \tag{3}$$

where n_i is the outward normal and $\tilde{u}_i$ and $\tilde{t}_i$ are the prescribed displacements and tractions, respectively. The initial conditions generally take the form of

$$\begin{aligned} u_i(\xi, 0) &= \mathring{u}_i(\xi), \quad \xi\in\Omega, \\ \dot{u}_i(\xi, 0) &= \mathring{v}_i(\xi), \quad \xi\in\Omega. \end{aligned} \tag{4}$$

For an integral formulation of an initial-boundary value problem, a convenient point of departure is Graffi's reciprocal theorem in elastodynamics (Wheeler and Sternberg, 1968). In terms of the Riemann convolution defined by

$$[g * h](\xi, t) = \begin{cases} 0, & \xi\in\Omega, \quad t < 0 \\ \displaystyle\int_0^t g(\xi, t-s)h(\xi, s)\,\mathrm{d}s, & \xi\in\Omega, \quad t > 0 \end{cases} \tag{5}$$

for a pair of functions g and h, the theorem can be stated as

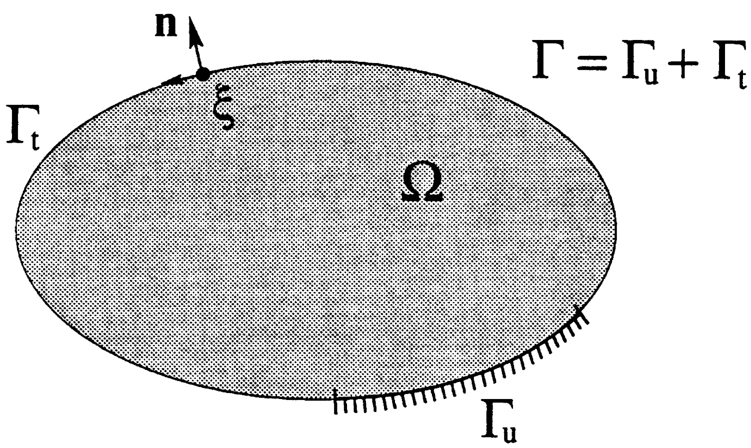

Fig. 1. Finite region: interior problem.

$$\int_{\Omega} f_i * \hat{u}_i \, \mathrm{d}\Omega_\xi + \int_{\Gamma} t_i * \hat{u}_i \, \mathrm{d}\Gamma_\xi - \int_{\Omega} \rho \ddot{u}_i * \hat{u}_i \, \mathrm{d}\Omega_\xi = \int_{\Omega} \hat{f}_i * u_i \, \mathrm{d}\Omega_\xi + \int_{\Gamma} \hat{t}_i * u_i \, \mathrm{d}\Gamma_\xi - \int_{\Omega} \rho \ddot{\hat{u}}_i * u_i \, \mathrm{d}\Omega_\xi \quad (6)$$

for two arbitrary elastodynamic states (u_i, τ_{ij}) and ($\hat{u}_i$, $\hat{\tau}_{ij}$) pertaining to the same solid medium. In terms of the initial data, eqn (6) can be reduced to

$$\int_{\Omega} f_i * \hat{u}_i \, \mathrm{d}\Omega_\xi + \int_{\Gamma} t_i * \hat{u}_i \, \mathrm{d}\Gamma_\xi + \int_{\Omega} \rho(\mathring{u}_i \hat{u}_i + \mathring{v}_i \dot{\hat{u}}_i) \, \mathrm{d}\Omega_\xi = \int_{\Omega} \hat{f}_i * u_i \, \mathrm{d}\Omega_\xi + \int_{\Gamma} \hat{t}_i * u_i \, \mathrm{d}\Gamma_\xi + \int_{\Omega} \rho(\mathring{\hat{u}}_i u_i + \mathring{\hat{v}}_i \dot{u}_i) \, \mathrm{d}\Omega_\xi \quad (7)$$

in the time domain. In the frequency domain, eqn (6) can be expressed even more compactly as

$$\int_{\Omega} F_i \hat{U}_i \, \mathrm{d}\Omega_\xi + \int_{\Gamma} T_i \hat{U}_i \, \mathrm{d}\Gamma_\xi = \int_{\Omega} \hat{F}_i U_i \, \mathrm{d}\Omega_\xi + \int_{\Gamma} \hat{T}_i U_i \, \mathrm{d}\Gamma_\xi \quad (8)$$

where the upper-case of an unknown denotes its Fourier transform with respect to the time, i.e.

$$G(\boldsymbol{\xi}, \omega) = \mathscr{F}[g(\boldsymbol{\xi}, t)] = \int_{-\infty}^{\infty} g(\boldsymbol{\xi}, t) e^{-i\omega t} \, \mathrm{d}t. \quad (9)$$

In view of its convenience in dealing with both elasticity and viscoelasticity problems whose formulations are analogous in terms of Fourier transforms by means of the correspondence principle (Christensen, 1971), the frequency-domain approach via eqn (8) will be employed as the analytical framework in this treatment.

To obtain an integral representation of an elastodynamic state in terms of boundary data, eqn (8) can be specialized by setting the body force $\hat{F}_i$ to be $\hat{F}_i^k$ which corresponds to a unit point load in the kth direction acting at a point $\boldsymbol{x} \in \Omega$, i.e.

$$\hat{F}_i^k(\xi, \omega) = \delta_{ik} \delta(\boldsymbol{x} - \boldsymbol{\xi}), \quad (10)$$

where δ_{ik} is the Kronecker delta and $\delta(\boldsymbol{x} - \boldsymbol{\xi})$ is the three-dimensional Dirac delta function. The corresponding Fourier transformed fundamental solutions $\hat{U}_i^k$, $\hat{T}_i^k$ and $\hat{\mathscr{T}}_{ij}^k$ which satisfy the field equations

$$(C_{ijpq} \hat{U}_{p,q}^k)_{,j} + \rho\omega^2 \hat{U}_i^k + \delta_{ik} \delta(\boldsymbol{x} - \boldsymbol{\xi}) = 0, \quad \hat{T}_i^k = \hat{\mathscr{T}}_{ij}^k n_j, \quad \hat{\mathscr{T}}_{ij}^k = C_{ijpq} \hat{U}_{p,q}^k, \quad (11)$$

are commonly called the displacement, traction and stress Green's functions, respectively. To emphasize their functional dependence, they will be written as $\hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)$, $\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n})$ and $\hat{\mathscr{T}}_{ij}^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)$; collectively, they describe the response at point $\boldsymbol{\xi}$ due to a time-harmonic unit point load acting in the kth direction at point $\boldsymbol{x}$ in Ω. By means of eqns (8)–(11), an integral representation of the displacement field in the interior of the region Ω (see Fig. 1) in terms of the boundary displacements and tractions on Γ bounding Ω, can be written as

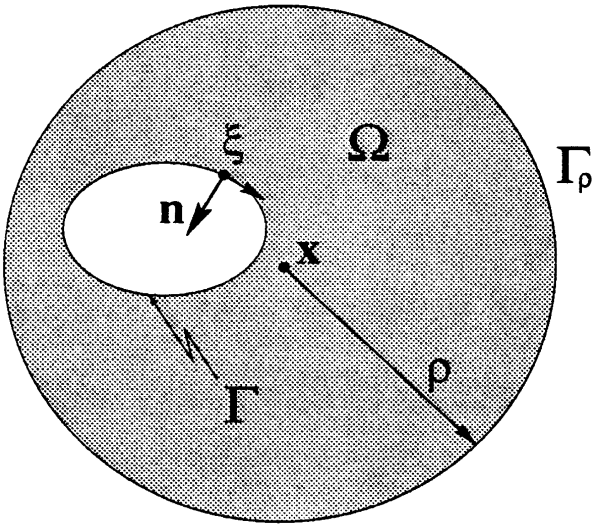

Fig. 2. Infinite region: exterior problem.

$$\mathscr{D}(\boldsymbol{x})U_k(\boldsymbol{x},\omega) = \int_\Gamma T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Gamma_\xi - \int_\Gamma U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi + \int_\Omega F_i(\boldsymbol{\xi},\omega)\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Omega_\xi, \quad \mathscr{D}(\boldsymbol{x}) = \begin{Bmatrix} 1, & \boldsymbol{x}\in\Omega \\ 0, & \boldsymbol{x}\notin\bar{\Omega} \end{Bmatrix} \tag{12}$$

in the frequency domain for a general anisotropic medium.

For an unbounded domain Ω which is exterior to Γ, the format of the integral representation in eqn (12) remains valid provided that (i) the direction of the outward normal for the exterior case is taken to be opposite to that for the interior case, and (ii) the solution satisfies

$$\lim_{\rho\to\infty}\int_{\Gamma_\rho}(T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega) - U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n}))\,\mathrm{d}\Gamma_\xi = 0, \quad \boldsymbol{x}\in\Omega \tag{13}$$

where Γ_ρ is the spherical outer surface with its radius $\rho\to\infty$ (see Fig. 2). For integral formulations, eqn (13) is essential as the generalized regularity condition for unbounded media.

3. Direct boundary integral equation methods

Fundamental to the mathematical derivation of boundary integral equation formulations is a clear understanding of the underlying limit process. To expose some of the intricacies involved, a review of the conventional boundary integral equation format is given in this section. The exposition also helps in setting the stage for an alternative general boundary integral equation formulation which involves only regular and weakly singular integrals that are amenable to analytical and numerical treatments.

3.1. Conventional direct boundary integral equation

To obtain the common limiting form of the fundamental integral eqn (12) as $\boldsymbol{x}$ goes to the boundary ($\boldsymbol{x} \to \boldsymbol{y} \in \Gamma$), one may view the boundary Γ as composed of two surfaces: one is a small surface region Γ_ε of radius ε, centered at the point $\boldsymbol{y}$; the other, $\Gamma - \Gamma_\varepsilon$, is the remainder of the boundary (see Fig. 3 for the case of a smooth surface). As a result of the foregoing decomposition, eqn (12) can be rewritten as

$$U_k(\boldsymbol{x},\omega) = \int_\Gamma T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Gamma_\xi - \int_{\Gamma-\Gamma_\varepsilon} U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi$$

$$-\int_{\Gamma_\varepsilon} U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi + \int_\Omega F_i(\boldsymbol{\xi},\omega)\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Omega_\xi, \quad \boldsymbol{x}\in\Omega, \qquad (14)$$

where $\boldsymbol{n}$ is the outward normal on Γ with respect to Ω. On account of the different orders of singularity of three-dimensional displacement and traction fundamental point-load solutions which behave as

$$\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega) = O\left(\frac{1}{|\boldsymbol{\xi}-\boldsymbol{x}|}\right) \quad \text{as } |\boldsymbol{\xi}-\boldsymbol{x}| \to 0,$$

$$\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n}) = O\left(\frac{1}{|\boldsymbol{\xi}-\boldsymbol{x}|^2}\right) \quad \text{as } |\boldsymbol{\xi}-\boldsymbol{x}| \to 0, \qquad (15)$$

respectively, the limiting form of eqn (14) as $\boldsymbol{x} \to \boldsymbol{y} \in \Gamma$ can be stated as

$$U_k(\boldsymbol{y},\omega) = \int_\Gamma T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Gamma_\xi - \lim_{\boldsymbol{x}\to\boldsymbol{y}} \int_{\Gamma-\Gamma_\varepsilon} U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi$$

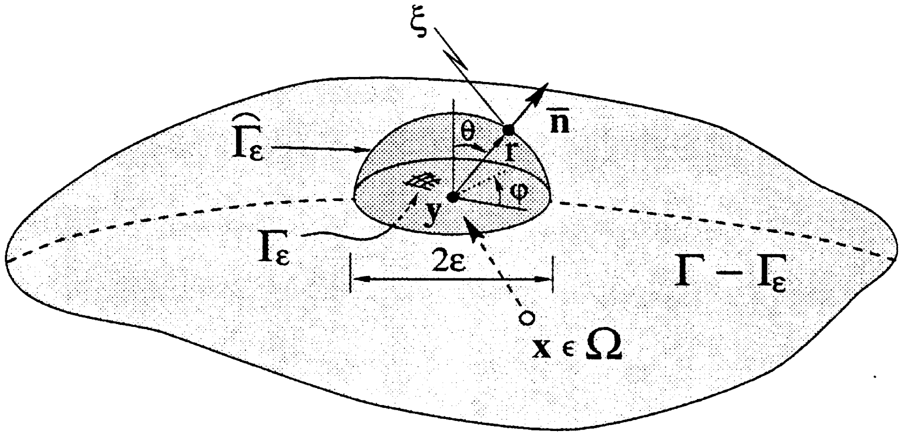

Fig. 3. Alternative domains of integration for c_{ik}.

$$-\lim_{\boldsymbol{x}\to\boldsymbol{y}}\int_{\Gamma_\varepsilon}[U_i(\boldsymbol{\xi},\omega)-U_i(\boldsymbol{y},\omega)]\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi$$

$$-U_i(\boldsymbol{y},\omega)\lim_{\boldsymbol{x}\to\boldsymbol{y}}\int_{\Gamma_\varepsilon}\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi+\int_\Omega F_i(\boldsymbol{\xi},\omega)\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Omega_\xi,\quad \boldsymbol{x}\in\Omega. \tag{16}$$

Under the hypothesis that the boundary traction and body-force distributions are sufficiently regular, the first and the last integrals on the right-hand side of eqn (16) exist in the ordinary sense as $\boldsymbol{x}\to\boldsymbol{y}\in\Gamma$ and can be evaluated numerically. As $\varepsilon\to 0$, the second integral on the right-hand side can be written as

$$\lim_{\varepsilon\to 0}\lim_{\boldsymbol{x}\to\boldsymbol{y}}\int_{\Gamma-\Gamma_\varepsilon}U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi\equiv \;⨍_\Gamma U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi \tag{17}$$

which is the definition of the Cauchy principal value of a surface integral. With the assumption that the displacement field is Hölder continuous, i.e.

$$U_i(\boldsymbol{\xi},\omega)=U_i(\boldsymbol{y},\omega)+O(|\boldsymbol{\xi}-\boldsymbol{y}|^\alpha) \tag{18}$$

where $0<\alpha\leqslant 1$, the limit of the third term in eqn (16) as $\varepsilon\to 0$ is

$$\lim_{\varepsilon\to 0}\lim_{\boldsymbol{x}\to\boldsymbol{y}}\int_{\Gamma_\varepsilon}[U_i(\boldsymbol{\xi},\omega)-U_i(\boldsymbol{y},\omega)]\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi=0 \tag{19}$$

as a result of eqn (15). By virtue of eqns (17) and (19), the limit of eqn (14) as $\boldsymbol{x}\to\boldsymbol{y}\in\Gamma$ can thus be expressed as

$$c_{ik}(\boldsymbol{y})U_i(\boldsymbol{y},\omega)=\int_\Gamma T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Gamma_\xi-⨍_\Gamma U_i(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi$$

$$+\int_\Omega F_i(\boldsymbol{\xi},\omega)\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Omega_\xi \tag{20}$$

where

$$c_{ik}(\boldsymbol{y})=\delta_{ik}+\lim_{\varepsilon\to 0}\left\{\lim_{\boldsymbol{x}\to\boldsymbol{y}}\int_{\Gamma_\varepsilon}\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi\right\},\quad \boldsymbol{y}\in\Gamma,\quad \boldsymbol{x}\in\Omega. \tag{21}$$

To clarify for eqn (21) the outcome of the series of limit operations whose order cannot be interchanged due to non-uniform convergence, it is useful to consider first the integral involved over an alternative closed surface $\Gamma_\varepsilon+\hat{\Gamma}_\varepsilon$, the latter of which is a partial spherical surface of radius ε, centered at $\boldsymbol{y}$ as depicted in Fig. 3 for the case of a smooth boundary. For $\boldsymbol{x}$ located in Ω and therefore outside the domain Ω_ε bounded by Γ_ε and $\hat{\Gamma}_\varepsilon$, the elastodynamic stress Green's functions $\hat{T}_{ij}^k(\boldsymbol{\xi},\boldsymbol{x},\omega)$ must satisfy the equations of motion

$$\hat{\mathscr{T}}_{ij,j}^k(\boldsymbol{\xi},\boldsymbol{x},\omega)=-\rho\omega^2\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega),\quad \boldsymbol{\xi}\in\Omega_\varepsilon,\quad \boldsymbol{x}\in\Omega. \tag{22}$$

With the definition of $\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \bar{\boldsymbol{n}}) = \hat{\mathscr{T}}_{ij}^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)\bar{n}_j(\boldsymbol{\xi})$ where $\bar{n}_j$ is the unit normal outward of Ω_ε, an application of the divergence theorem yields

$$\int_{\Omega_\varepsilon} \hat{\mathscr{T}}_{ij,j}^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)\, \mathrm{d}\Omega_\xi = \int_{\Gamma_\varepsilon + \hat{\Gamma}_\varepsilon} \hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \bar{\boldsymbol{n}})\, \mathrm{d}\Gamma_\xi, \tag{23}$$

which in turn implies

$$\int_{\Gamma_\varepsilon} \hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \bar{\boldsymbol{n}})\, \mathrm{d}\Gamma_\xi = -\int_{\hat{\Gamma}_\varepsilon} \hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \bar{\boldsymbol{n}})\, \mathrm{d}\Gamma_\xi - \rho\omega^2 \int_{\Omega_\varepsilon} \hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)\, \mathrm{d}\Omega_\xi. \tag{24}$$

By virtue of the regularity of $\hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)$ in Ω_ε, it can be shown that

$$\lim_{\varepsilon \to 0} \left\{ \lim_{\boldsymbol{x} \to \boldsymbol{y}} \int_{\Omega_\varepsilon} \hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)\, \mathrm{d}\Omega_\xi \right\} = 0. \tag{25}$$

With the aid of eqn (24) and the identity

$$\bar{\boldsymbol{n}}(\xi) = -\boldsymbol{n}(\xi), \quad \xi \in \Gamma_\varepsilon,$$

where $\boldsymbol{n}$ is the outward normal external to Ω, eqn (21) can be reduced to

$$c_{ik}(\boldsymbol{y}) = \delta_{ik} + \lim_{\varepsilon \to 0} \int_{\hat{\Gamma}_\varepsilon} \hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \bar{\boldsymbol{n}})\, \mathrm{d}\Gamma_\xi, \quad \boldsymbol{y} \in \Gamma \tag{26}$$

whose integrand $\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \bar{\boldsymbol{n}})$ will stay non-singular for $\boldsymbol{\xi} \in \hat{\Gamma}_\varepsilon$ in the remaining limit process.

For further simplification, it should be noted that the point-load Green's function can be decomposed into a singular part ($[\hat{U}_i^k]_1$, $[\hat{\mathscr{T}}_{ij}^k]_1$) and a regular part ($[\hat{U}_i^k]_2$, $[\hat{T}_{ij}^k]_2$) such that

$$\begin{aligned} \hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega) &= [\hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)]_1 + [\hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega)]_2, \\ \hat{\mathscr{T}}_{ij}^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n}) &= [\hat{\mathscr{T}}_{ij}^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n})]_1 + [\hat{\mathscr{T}}_{ij}^k(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n})]_2, \end{aligned} \tag{27}$$

the second of which, in turn, defines the corresponding singular part $[\hat{T}_i^k]_1$ and the regular part $[\hat{T}_i^k]_2$ of the traction Green's function. As only $[\hat{T}_i^k]_1$ will contribute to the integral over $\hat{\Gamma}_\varepsilon$ in eqn (26) as $\varepsilon \to 0$, eqn (26) can be reduced to

$$c_{ij}(\boldsymbol{y}) = \delta_{ik} + \lim_{\varepsilon \to 0} \int_{\hat{\Gamma}_\varepsilon} [\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \bar{\boldsymbol{n}})]_1\, \mathrm{d}\Gamma_\xi, \quad \boldsymbol{y} \in \Gamma, \tag{28}$$

which involves only the singular part of the traction Green's function. To extract totally the singularity of the point-load Green's function for all possible source and observation points in a piecewise homogeneous, isotropic, multi-layered solid, a case of common interest in many practical applications, it can be shown by asymptotic analysis that the simplest choice for the singular part $[\hat{T}_i^k]_1$ of the Green's function without any loss of generality is the static bi-material full-space Green's function (Guzina and Pak, 1998) whose plane of material discontinuity coincides with the nearest material interface in the multi-layered system. As will be illustrated later, the employment of neither the Kelvin's state, i.e.

$$[\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)]_1 = \frac{1}{16\pi(1-\nu)\mu r}\{(3-4\nu)\delta_{ik}+r_{,i}r_{,k}\},$$

$$[\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})]_1 = \frac{-1}{8\pi(1-\nu)r^2}\{((1-2\nu)\delta_{ik}+3r_{,i}r_{,k})(r_{,m}n_m) - (1-2\nu)(r_{,k}n_i-r_{,i}n_k)\},$$

$$r = |\boldsymbol{\xi}-\boldsymbol{x}|, \tag{29}$$

nor its half-space counterpart, Mindlin's solution, is satisfactory for such purposes. For a smooth Γ in a homogeneous and isotropic solid, it can be shown using spherical coordinates that

$$\lim_{\varepsilon\to 0}\int_{\hat{\Gamma}_\varepsilon}[\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\bar{\boldsymbol{n}})]_1\,\mathrm{d}\Gamma_\xi = -\frac{\delta_{ik}}{2}, \quad i,k=1,2,3, \quad \boldsymbol{y}\in\Gamma \tag{30}$$

so that $c_{ik}(\boldsymbol{y}) = \delta_{ik}/2$. For non-smooth boundary points in such a medium, closed-form results for c_{ik} in three-dimensional problems can also be derived (see Hartmann, 1982). Equations (20) and (26) constitute the conventional direct boundary integral equation formulation in solid and geomechanics.

3.2. *Alternative direct boundary integral equation*

Despite its traditional appeal, the direct boundary integral equation formulation, in terms of eqns (20) and (26), is not free from some perennial objections. For instance, the second integral on the right-hand side of eqn (20) is defined in terms of its Cauchy principal value whose computation requires techniques beyond ordinary quadrature methods (see Lachat and Watson, 1976). For nonhomogeneous media and non-smooth boundary geometries, direct evaluations of the coefficients c_{ik} in eqn (28) can also pose considerable analytical difficulties. In this section, it will be shown that an alternative form of the boundary integral equation can be developed which is equally compact but void of such extraneous complexities. Such a reduction is apt to make direct boundary integral equation methods more palatable to both engineers and mechanicians.

To begin, it is useful to note that for both interior and exterior probe the fundamental integral representation (12) of the response at an interior point may be re-arranged as

$$\int_\Gamma T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Gamma_\xi - \int_\Gamma [U_i(\boldsymbol{\xi},\omega)-U_i(\boldsymbol{x},\omega)]\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi$$

$$- U_i(\boldsymbol{x},\omega)\int_\Gamma \hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi + \int_\Omega F_i(\boldsymbol{\xi},\omega)\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Omega_\xi$$

$$= U_k(\boldsymbol{x},\omega), \quad \boldsymbol{x}\in\Omega. \tag{31}$$

Furthermore, in the proposed decomposition of the dynamic point-load Green's functions ($\hat{U}_i^k$, $\hat{\mathcal{T}}_{ij}^k$) into a static part ($[\hat{U}_i^k]_1$, $[\hat{\mathcal{T}}_{ij}^k]_1$) and a residual ($[\hat{U}_i^k]_2$, $[\hat{\mathcal{T}}_{ij}^k]_2$) (see eqn (27)), the singular and regular parts have the properties such that

$$[\hat{\mathscr{T}}^k_{ij,j}]_1 + \delta_{ik}\delta(\boldsymbol{x}-\boldsymbol{\xi}) = 0, \tag{32}$$

and

$$[\hat{\mathscr{T}}^k_{ij,j}]_2 + \rho\omega^2 \hat{U}^k_i = 0. \tag{33}$$

A consequence of eqn (32) is that

$$\int_\Gamma [\hat{T}^k_i(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n})]_1 \, \mathrm{d}\Gamma_\xi = \begin{Bmatrix} -\delta_{ik}, & \boldsymbol{x} \in \Omega & \text{(Int. Problem)} \\ 0, & \boldsymbol{x} \in \Omega & \text{(Ext. Problem)} \end{Bmatrix}. \tag{34}$$

On account of eqns (32) to (34), eqn (31) can be reduced to

$$\int_\Gamma T_i(\boldsymbol{\xi}, \omega; \boldsymbol{n}) \hat{U}^k_i(\boldsymbol{\xi}, \boldsymbol{x}, \omega) \, \mathrm{d}\Gamma_\xi - \int_\Gamma [U_i(\boldsymbol{\xi}, \omega) - U_i(\boldsymbol{x}, \omega)][\hat{T}^k_i(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n})]_1 \, \mathrm{d}\Gamma_\xi$$
$$- \int_\Gamma U_i(\boldsymbol{\xi}, \omega)[\hat{T}^k_i(\boldsymbol{\xi}, \boldsymbol{x}, \omega; \boldsymbol{n})]_2 \, \mathrm{d}\Gamma_\xi + \int_\Omega F_i(\boldsymbol{\xi}, \omega) \hat{U}^k_i(\boldsymbol{\xi}, \boldsymbol{x}, \omega) \, \mathrm{d}\Omega_\xi$$
$$= \begin{Bmatrix} 0 & \text{(Int. Problem)} \\ 1 & \text{(Ext. Problem)} \end{Bmatrix} U_k(\boldsymbol{x}, \omega), \quad \boldsymbol{x} \in \Omega. \tag{35}$$

Provided that the displacement $U_i(\boldsymbol{x}, \omega)$ satisfies the Hölder continuity condition and the traction $\boldsymbol{T}_i(\boldsymbol{x}, \omega; \boldsymbol{n})$ is not too singular on the boundary, all the integrands in eqn (35) will be at most weakly singular as $\boldsymbol{x} \to \boldsymbol{y} \in \Gamma$ in the integrations intended. Accordingly, one can write, with reference to the Green's function decomposition in eqn (27), that

$$\int_\Gamma T_i(\boldsymbol{\xi}, \omega; \boldsymbol{n}) \hat{U}^k_i(\boldsymbol{\xi}, \boldsymbol{y}, \omega) \, \mathrm{d}\Gamma_\xi - \int_\Gamma [U_i(\boldsymbol{\xi}, \omega) - U_i(\boldsymbol{y}, \omega)][\hat{T}^k_i(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})]_1 \, \mathrm{d}\Gamma_\xi$$
$$- \int_\Gamma U_i(\boldsymbol{\xi}, \omega)[\hat{T}^k_i(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})]_2 \, \mathrm{d}\Gamma_\xi + \int_\Omega F_i(\boldsymbol{\xi}, \omega) \hat{U}^k_i(\boldsymbol{\xi}, \boldsymbol{y}, \omega) \, \mathrm{d}\Omega_\xi$$
$$= \begin{Bmatrix} 0 & \text{(Int. Problem)} \\ 1 & \text{(Ext. Problem)} \end{Bmatrix} U_k(\boldsymbol{y}, \omega), \quad \boldsymbol{y} \in \Gamma. \tag{36}$$

Equation (36) represents a direct boundary integral equation formulation which is free of (i) Cauchy principal values and (ii) the need of the coefficients c_{ik}. Because of such features, its numerical implementation is straightforward, requiring only a minor re-arrangement of terms in a conventional code. As a refined analog of Sladek and Sladek (1991), the regularized integral format can be generalized to tackle seismic wave excitation problems in earthquake engineering as will be shown in the next section. With its analytical simplicity, the integral eqn (36) is preferable over eqn (20) as the mathematical foundation for developing rigorous treatments of complicated singular mixed boundary value problems (Sternberg, 1980) such as those involving sharp body geometries and material discontinuities in geomechanics and soil-structure contact problems. For completeness, its connection with the conventional boundary integral eqn (20) can also be recognized by virtue of eqns (17), (26), (34) and the identity

$$c_{ik}(\boldsymbol{y}) + \int_{\Gamma} \hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})\, \mathrm{d}\Gamma_\xi = \delta_{ik} + \lim_{\varepsilon \to 0} \int_{\Gamma - \Gamma_\varepsilon + \hat{\Gamma}_\varepsilon} \hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})\, \mathrm{d}\Gamma_\xi$$

$$= \int_{\Gamma} [\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})]_2\, \mathrm{d}\Gamma_\xi + \left\{ \begin{matrix} 0 & \text{(Int. Problem)} \\ 1 & \text{(Ext. Problem)} \end{matrix} \right\} \delta_{ik}, \quad \boldsymbol{y} \in \Gamma. \tag{37}$$

In engineering applications where the body-force field can be ignored or incorporated separately, the governing boundary integral equation in eqn (36) further simplifies to

$$\int_{\Gamma} T_i(\boldsymbol{\xi}, \omega; \boldsymbol{n}) \hat{U}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega)\, \mathrm{d}\Gamma_\xi - \int_{\Gamma} [U_i(\boldsymbol{\xi}, \omega) - U_i(\boldsymbol{y}, \omega)][\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})]_1\, \mathrm{d}\Gamma_\xi$$

$$- \int_{\Gamma} U_i(\boldsymbol{\xi}, \omega)[\hat{T}_i^k(\boldsymbol{\xi}, \boldsymbol{y}, \omega; \boldsymbol{n})]_2\, \mathrm{d}\Gamma_\xi = \left\{ \begin{matrix} 0 & \text{(Int. Problem)} \\ 1 & \text{(Ext. Problem)} \end{matrix} \right\} U_k(\boldsymbol{y}, \omega), \quad \boldsymbol{y} \in \Gamma \tag{38}$$

which is the framework employed in the following treatment.

4. Application to seismic soil-structure interaction

4.1. Multi-layered viscoelastic half-space Green's functions

To incorporate the important effects caused by in situ variations of soil parameters and stratifications of soil media, a complete set of displacement and stress Green's functions for an isotropic, piecewise-homogeneous, multi-layered, viscoelastic half-space has been developed for earthquake engineering and soil dynamics applications (Guzina, 1996). In contrast to past treatments (e.g. Luco and Apsel 1983; Apsel and Luco, 1983), the mathematical solution is derived by the propagator-matrix approach (Thomson, 1950; Haskell, 1953) in conjunction with a method of potentials and Hankel transforms (Pak, 1987), leading to a concise and efficient approach to this class of complicated elastodynamic boundary value problems. More importantly, with the aid of a set of dual representations for the static bi-material full-space Green's functions (Guzina and Pak, 1998), a systematic procedure is established by which the singular part of the integral representations of the dynamic multi-layered Green's functions can always be extracted completely. Such a possibility is central to an accurate numerical evaluation of the fundamental solutions as well as the theoretical foundation of the regularized direct boundary integral equation method. By a judicious contour integration on the complex wave-number plane, the oscillations of the residual Green's function integrands can be greatly reduced (Pak, 1987) while the exact limit of purely elastic soil behavior, an important theoretical benchmark, can be realized with no difficulty. To illustrate the fundamental importance of the proposed Green's function decomposition in a rigorous treatment, the difference between the time-harmonic vertical displacement Green's function for a unit vertical point load in the interior of a three-layer elastic half-space (see Fig. 4) and the pertinent static bi-material solution along the z-axis is shown in Fig. 5 as an example. From the plot, one can see that the residual Green's function after the proposed extraction is indeed smooth and finite, with no singularity anywhere. This is in contrast to the performance shown in the same figure of the common scheme of subtracting from the Green's function the Kelvin's or Mindlin's state with a

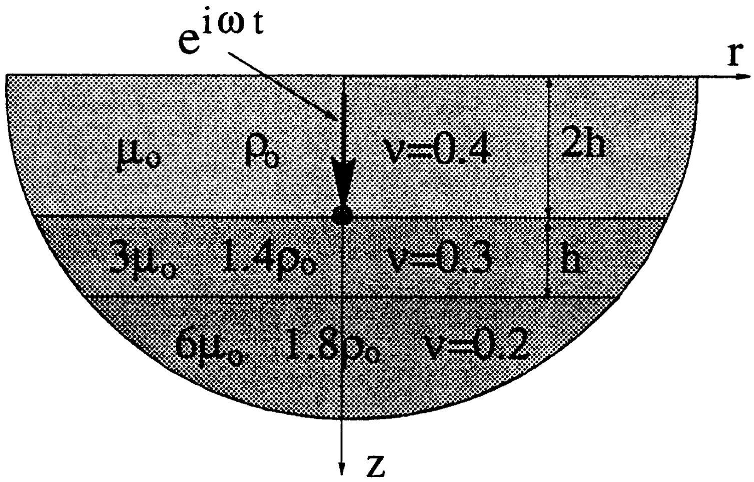

Fig. 4. An axial point load in a three-layer half-space.

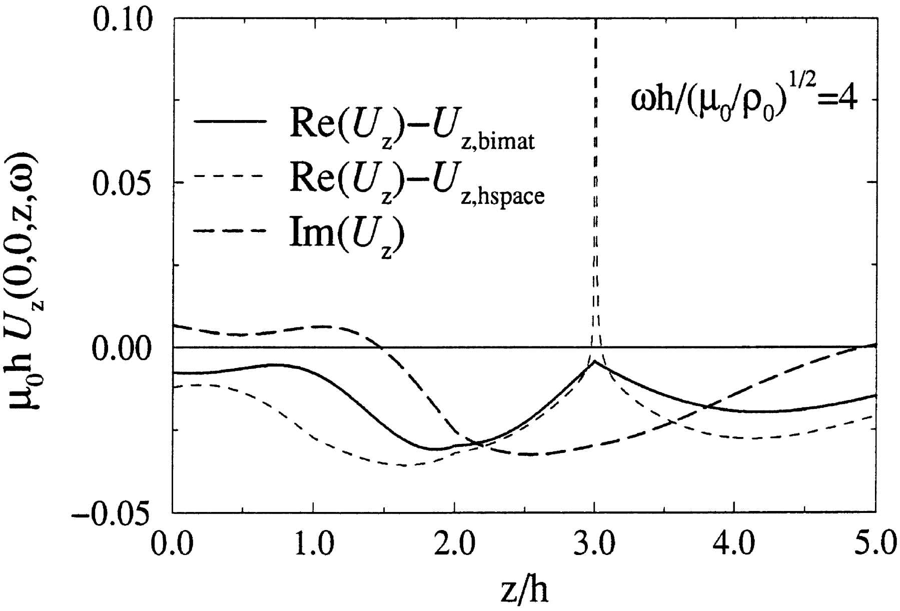

Fig. 5. Singularity extraction for dynamic Green's function.

locally averaged shear modulus and Poisson's ratio. As is evident from the display, a serious residual singularity persists in the Green's function despite such an extraction, demonstrating the inadequacy of the popular method.

The advantages of employing Green's functions for the half-space geometry to represent the

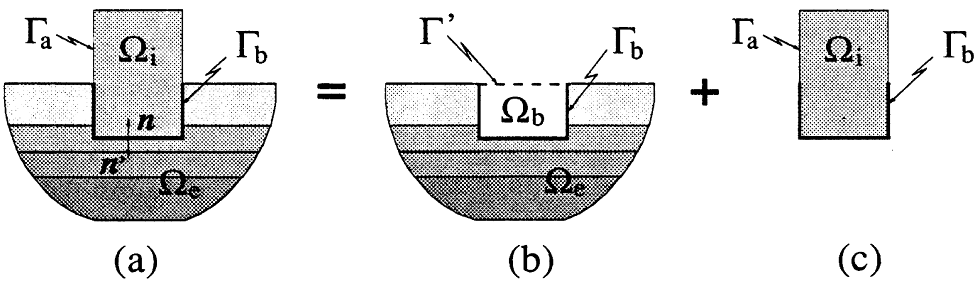

Fig. 6. Domain decomposition of soil-structure-foundation systems.

semi-infinite open soil region Ω_e in the generic soil-structure interaction problem shown in Fig. 6 can be easily seen by considering the boundary Γ in eqn (38) for exterior problems as composed of $\Gamma_b+\Gamma'$, where Γ_b is the soil-foundation interface and Γ' is the part of the free surface of the half-space closing out the open region Ω_b (see Fig. 6b). As the boundary Γ' can be regarded as fictitious, one can require that the traction-free condition

$$T_i(\boldsymbol{\xi},\omega;\boldsymbol{n}(\boldsymbol{\xi}))=0,\quad \boldsymbol{\xi}\in\Gamma' \tag{39}$$

be satisfied. Owing to the fact that the traction Green's functions for the half-space also vanish on Γ', i.e.

$$\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n}(\boldsymbol{\xi}))=0,\quad \boldsymbol{\xi}\in\Gamma', \tag{40}$$

the regularized direct boundary integral eqn (38) for the soil domain reduces to

$$\int_{\Gamma_b} T_i(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Gamma_\xi-\int_{\Gamma_b}[U_i(\boldsymbol{\xi},\omega)-U_i(\boldsymbol{y},\omega)][\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})]_1\,\mathrm{d}\Gamma_\xi$$
$$-\int_{\Gamma_b} U_i(\boldsymbol{\xi},\omega)[\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})]_2\,\mathrm{d}\Gamma_\xi=\mathscr{B}_{ik}(\boldsymbol{y},\omega)U_i(\boldsymbol{y},\omega),\quad \boldsymbol{y}\in\Gamma_b \tag{41}$$

where

$$\mathscr{B}_{ik}(\boldsymbol{y},\omega)=\delta_{ik}-\int_{\Gamma'}[\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})]_1\,\mathrm{d}\Gamma_\xi,\quad \boldsymbol{y}\in\Gamma_b. \tag{42}$$

As can be seen from the governing boundary integral eqn (41), the formulation using half-space Green's functions involves quantities on the soil-foundation interface Γ_b only. Furthermore, the integral in eqn (42) needs to be evaluated only if the singular part of the traction Green's function does not satisfy the traction-free condition on Γ'. In using the dynamic fundamental solution for a piecewise homogeneous multi-layered half-space, the singular part of its traction Green's functions at any material point is taken, as mentioned earlier, to be the static bi-material full-space Green's function with its plane of material discontinuity aligned with the nearest solid interface in the multi-layered medium. By virtue of this definition, the singular part of the Green's function will degenerate to Mindlin's solution if the nearest interface is the free-surface. As a result, the integral on the right-hand side of eqn (42) is possibly nonzero only for collocation points $\boldsymbol{y}$ which are sufficiently far from Γ'. As such, the integrand in eqn (42) will not be singular and may be efficiently integrated by regular quadrature.

To treat the finite domain Ω_i of the embedded foundation or structure in Fig. 6c as a continuum (e.g. in the case of a dam), the boundary integral equation format in eqn (38) for interior problems can be employed with the boundary surface identified as $\Gamma_a + \Gamma_b$. With the outward normals to the excavated half-space and the embedment denoted as $\boldsymbol{n}$ and $\boldsymbol{n}'$, respectively (see Fig. 6a), the interfacial displacement and traction compatibility conditions on Γ_b may be written as

$$U_i(\boldsymbol{y},\omega) = V_i(\boldsymbol{y},\omega), \quad T_i(\boldsymbol{y},\omega,\boldsymbol{n}) = -S_i(\boldsymbol{y},\omega,\boldsymbol{n}'), \quad \boldsymbol{y}\in\Gamma_b. \tag{43}$$

In the above, U_i and T_i are the boundary displacements and tractions of the soil region Ω_e, while V_i and S_i are the boundary displacements and tractions for the structure/foundation domain Ω_i. As will be seen in the next section for seismic excitations, a system of boundary integral equations involving unknowns on only Γ_a and Γ_b can be formulated for general continuum soil-structure interaction problems.

4.2. Scattering problems

To ensure the generalized regularity condition (13), is satisfied so that the boundary integral equation formulation in eqns (38) or (41) is valid, the source of dynamic excitation must be located in a finite subdomain in the infinite exterior region. For scattering problems such as those involving incident seismic waves from infinity, however, the total solution would clearly violate the regularity condition (13). To circumvent the difficulty, the approach of decomposing the total displacement field $U_i^*(\boldsymbol{\xi},\omega)$ in a seismic problem into a free field U_i^F and a scattered field U_i^S such that

$$U_i^*(\boldsymbol{\xi},\omega) = U_i^F(\boldsymbol{\xi},\omega) + U_i^S(\boldsymbol{\xi},\omega) \tag{44}$$

can be employed where $U_i^F(\boldsymbol{\xi},\omega)$ is defined to be the response of the corresponding unexcavated half-space due to the prescribed incident body or surface seismic waves (see e.g. Niwa et al., 1986). In the case of a homogeneous medium, the free-field motion is composed of the incident waves and those reflected by the free-surface. In the case of a multi-layered half-space, however, it includes the multiple reflections and refractions caused by the free surface and all internal layer interfaces. With such definitions, the free field and the scattered field must therefore satisfy the Navier equations with zero body forces independently. With the scattered field satisfying the generalized regularity condition (13) as an additional consequence, an integral representation for U_i^S in the soil domain can be written as

$$\mathscr{D}(\boldsymbol{x})\,U_k^S(\boldsymbol{x},\omega) = \int_{\Gamma_b} T_i^S(\boldsymbol{\xi},\omega;\boldsymbol{n})\,\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_b} U_i^S(\boldsymbol{\xi},\omega)\,\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi, \quad \mathscr{D}(\boldsymbol{x}) = \begin{Bmatrix} 1, & \boldsymbol{x}\in\Omega_e \\ 0, & \boldsymbol{x}\in\Omega_b \end{Bmatrix}, \tag{45}$$

where $\hat{U}_i^k$ and $\hat{T}_i^k$ are the displacement and traction Green's functions for a multi-layered half-space as in (41), and T_i^S is the traction field associated with the scattered motion. Owing to the fact that the free field satisfies the Navier equations of motion in the unexcavated half-space, one can also use the integral representation for internal-domain problems to write

$$\mathscr{D}(\boldsymbol{x})U_k^F(\boldsymbol{x},\omega) = \int_{\Gamma_b} T_i^F(\boldsymbol{\xi},\omega;\boldsymbol{n}')\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_b} U_i^F(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n}')\,\mathrm{d}\Gamma_\xi, \quad \mathscr{D}(\boldsymbol{x}) = \begin{Bmatrix} 0, & \boldsymbol{x}\in\Omega_e \\ 1, & \boldsymbol{x}\in\Omega_b \end{Bmatrix}. \tag{46}$$

By virtue of eqn (44) and the identities

$$\boldsymbol{n}'(\boldsymbol{\xi}) = -\boldsymbol{n}(\boldsymbol{\xi}), \quad T_i^S(\boldsymbol{\xi},\omega;\boldsymbol{n}') = -T_i^S(\boldsymbol{\xi},\omega;\boldsymbol{n}), \quad \boldsymbol{\xi}\in\Gamma_b. \tag{47}$$

Equations (45) and (46) may be subtracted from each other, since they involve integrals over the same boundary domain Γ_b. As a result, an integral representation of the total field in the excavated half-space Ω_e and its complement Ω_b can be expressed as

$$\mathscr{D}(\boldsymbol{x})U_k^*(\boldsymbol{x},\omega) = U_k^F(\boldsymbol{x},\omega) + \int_{\Gamma_b} T_i^*(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega)\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_b} U_i^*(\boldsymbol{\xi},\omega)\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{x},\omega;\boldsymbol{n})\,\mathrm{d}\Gamma_\xi, \quad \mathscr{D}(\boldsymbol{x}) = \begin{Bmatrix} 1, & \boldsymbol{x}\in\Omega_e \\ 0, & \boldsymbol{x}\in\Omega_b \end{Bmatrix} \tag{48}$$

in terms of the half-space Green's functions.

Finally, upon taking the limit $\boldsymbol{x}\to\boldsymbol{y}\in\Gamma_b$ with $\boldsymbol{x}\in\Omega_e$, one gets a direct boundary integral equation for the exterior domain Ω_e in terms of the total displacement field U_i^* and traction field T_i^* in

$$\int_{\Gamma_b} T_i^*(\boldsymbol{\xi},\omega;\boldsymbol{n})\hat{U}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_b}[U_i^*(\boldsymbol{\xi},\omega)-U_i^*(\boldsymbol{y},\omega)][\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})]_1\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_b} U_i^*(\boldsymbol{\xi},\omega)[\hat{T}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n})]_2\,\mathrm{d}\Gamma_\xi + U_k^F(\boldsymbol{y},\omega) = \mathscr{B}_{ik}(\boldsymbol{y},\omega)U_k^*(\boldsymbol{y},\omega), \quad \boldsymbol{y}\in\Gamma_b, \tag{49}$$

where $\mathscr{B}_{ik}$ is defined by eqn (42). One may observe that the only difference of eqn (49) from the boundary integral eqn (41) is the free-field term on the left-hand side for the seismic problem.

In treating the structural/foundation domain Ω_i as a continuum, the interior format of the boundary integral equation formulation in eqn (38) can be used, leading to

$$\int_{\Gamma_a+\Gamma_b} S_i^*(\boldsymbol{\xi},\omega;\boldsymbol{n}')\hat{V}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega)\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_a+\Gamma_b}[V_i^*(\boldsymbol{\xi},\omega)-V_i^*(\boldsymbol{y},\omega)][\hat{S}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n}')_1\,\mathrm{d}\Gamma_\xi - \int_{\Gamma_a+\Gamma_b} V_i^*(\boldsymbol{\xi},\omega)[\hat{S}_i^k(\boldsymbol{\xi},\boldsymbol{y},\omega;\boldsymbol{n}')]_2\,\mathrm{d}\Gamma_\xi = 0, \quad \boldsymbol{y}\in\Gamma_a+\Gamma_b \tag{50}$$

where $\hat{V}_i^k$ and $\hat{S}_i^k$ can be taken to be the pertinent full-space displacement and traction Green's functions for the domain Ω_i. Boundary integral eqns (49) and (50) are coupled through the interfacial conditions in eqn (43) and can be solved numerically upon discretization. For the case of a discrete structural domain such as a building, eqn (50) can be replaced by a dynamic stiffness matrix representation which may permit a further reduction in the degrees of freedom.

4.3. Singular contact tractions

Typical in all common soil-foundation configurations is the presence of sharp geometries such as corners and edges. The result is the theoretical occurrence of singular contact tractions at the soil-foundation interfaces which can create inaccuracy and poor convergence in the numerical analysis. To improve the performance of boundary integral equation methods, the incorporation of special elements capable of representing singular tractions would thus be desirable. Such an approach dates back to Cruse and Wilson (1977) on the application of boundary integral equation methods to linear fracture mechanics. A recent example is the continuous eight-node quadratic element with one square-root singular edge (Luchi and Rizzuti, 1993) for three-dimensional crack analysis. Aggravating the situation further, however, is the fact that the boundary or interfacial tractions are also non-unique at such abrupt geometric locations. Such a problem can be alleviated by the use of double nodes, tangential derivatures and Hooke's law, or discontinuous boundary elements (e.g. Rego Silva et al., 1993; Stamos and Beskos, 1995). To further elevate the performance and flexibility of boundary integral equation methods in handling the foregoing aspects simultaneously in three-dimensional elastodynamics and elastostatics, a set of four-node semi-discontinuous singular edge and corner elements is developed (Guzina, 1996) for modeling such boundary tractions. For instance, the traction shape functions for the singular edge element are defined to be

$$\phi_1^t(\eta_1,\eta_2) = \frac{2^{1-\kappa_1}}{\mathscr{R}_e}(1-\eta_2)\{-(1-c)^{\kappa_1}(1-\eta_1)^{\kappa_1-1}+(1-\eta_1)^{2\kappa_1-1}\},$$

$$\phi_2^t(\eta_1,\eta_2) = \frac{(1-c)^{1-\kappa_1}}{\mathscr{R}_e}(1-\eta_2)\{2^{\kappa_1}(1-\eta_1)^{\kappa_1-1}-(1-\eta_1)^{2\kappa_1-1}\},$$

$$\phi_3^t(\eta_1,\eta_2) = \frac{(1-c)^{1-\kappa_1}}{\mathscr{R}_e}(1+\eta_2)\{2^{\kappa_1}(1-\eta_1)^{\kappa_1-1}-(1-\eta_1)^{2\kappa_1-1}\},$$

$$\phi_4^t(\eta_1,\eta_2) = \frac{2^{1-\kappa_1}}{\mathscr{R}_e}(1+\eta_2)\{-(1-c)^{\kappa_1}(1-\eta_1)^{\kappa_1-1}+(1-\eta_1)^{2\kappa_1-1}\}, \tag{51}$$

$$\mathscr{R}_e = 2(2^{\kappa_1}-(1-c)^{\kappa_1}), \tag{52}$$

with reference to the parent element domain in Fig. 7. In contrast to the singular elements used in conventional fracture mechanics where only the square-root singularity is of interest (e.g. see Luchi and Rizzuti, 1993), the constructed traction shape functions can be used to represent power-type singularities of any order via a suitable choice of the parameters κ_1 and c. Apart from being capable of dealing with arbitrary power-type surface traction singularities and discontinuities across boundary elements, these elements also permit a smooth transition to standard bilinear quadrilateral elements away from the edges. Together with a corresponding family of non-singular isoparametric displacement and geometric shape functions, the singular surface elements have been implemented into a computational platform for the regularized boundary integral equation

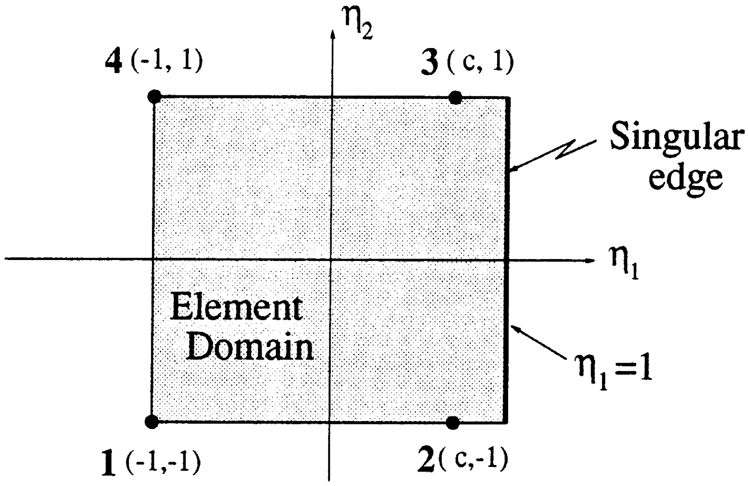

Fig. 7. Singular edge element: parent domain.

method on the basis of Gaussian quadrature. Enhanced by the foregoing analytical and computational developments, the regularized direct boundary integral equation method is found to perform very well, with fast and stable convergence in both primary and secondary variables. To illustrate the performance of the composite scheme, selected numerical results are presented in the next section.

5. Numerical results

By means of the regularized boundary integral equation method discussed, a set of benchmark solutions have been generated for some fundamental soil-structure interaction problems associated with a rigid block foundation with arbitrary embedment under forced and seismic wave excitations. To provide a full picture of the response, typical results of the foundation-soil contact tractions, dynamic impedances for surface and embedded foundations, as well as the seismic input-motion functions will be given as illustrations. For reference, the key notations in the soil-structure interaction problem are defined in Fig. 8.

As a degenerate case of the foundation configuration shown in Fig. 8, the problem of a square rigid surface foundation of dimension $2a \times 2a$ with $h = 0$ which is fully bonded to a homogeneous isotropic elastic half-space with a shear modulus μ, mass density ρ and Poisson's ratio $v = 1/3$ is first examined. Under the fully-bonded interfacial condition, the contact traction singularity at the foundation edges is of the square-root type which is accounted for through the use of the singular boundary elements discussed in the last section. An example of the contact shear stress distribution beneath the footing undergoing a time-harmonic pure horizontal translation Δ_x at a dimensionless frequency $\bar{\omega} = \omega a/\sqrt{\mu/\rho} = 2.5$ is shown in Fig. 9 where the prominence of the singularities at the edge and corner regions is clearly displayed. The corresponding foundation impedance K_{hh} is shown in Figs 10 and 11 in terms of the normalized horizontal stiffness and damping coefficients k_{hh} and c_{hh} which are defined by

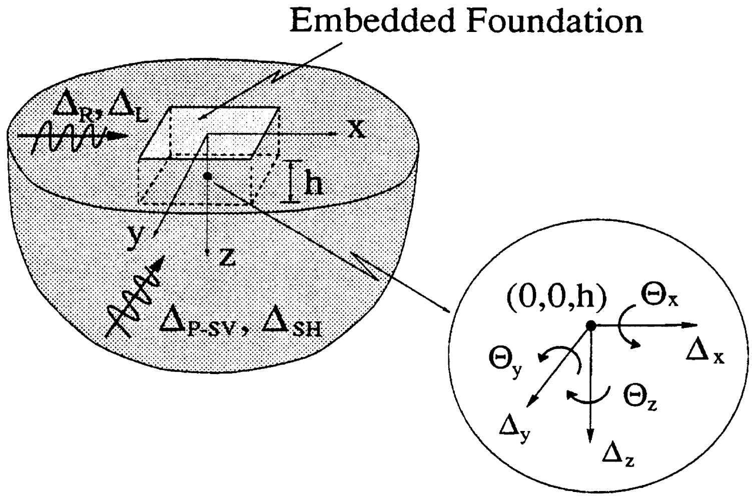

Fig. 8. Geometry of embedded foundation and coordinate system.

$$K_{hh}(\varpi) = \mu a\{k_{hh}(\varpi) + i\varpi c_{hh}(\varpi)\}. \tag{53}$$

Included in the figures are the results obtained by the indirect boundary element methods in (i) Wong and Luco (1985) who employ a piecewise constant stress distribution (see Mita and Luco (1989) for tabulation), and (ii) Triantafyllidis (1986) whose formulation recognizes the singularity of the contact stresses but demands numerical computation of difficult improper double integrals. In the example, the performance of Wong and Luco (1985) is apparently better than Triantafyllidis (1986).

The next example is the case of an embedded, massless, rigid block foundation, with dimension $2a \times 2a$ with $h = a$, bonded to a homogeneous viscoelastic half-space with a complex shear modulus

$$\mu = \mu_0(1+2i\zeta) = \mu_0(1+0.002i) \tag{54}$$

and a Poisson's ratio $v = 1/3$. The parameter $\zeta = 0.001$ is the hysteretic damping ratio which is taken to be common for both compressional and shear waves. The resulting dynamic foundation impedances K_{hh} and K_{mm} are shown in Figs 12 and 13 with reference to the center of the foundation base $(0, 0, h)$ in terms of the stiffness and damping coefficients in

$$K_{mm}(\varpi) = \mu a^3 \bar{K}_{mm}(\varpi) = \mu a^3\{k_{mm}(\varpi) + i\varpi c_{mm}(\varpi)\}. \tag{55}$$

For comparison, the result of Mita and Luco (1989) who employ an off-boundary source-collocation scheme together with a finite element discretization for a similar viscoelastic half-space with a slightly complex Poisson's ratio of $v = 1/3 - 0.00017\,i$ is also included.

By means of the direct boundary integral eqn (49) for seismic wave problems, the foundation input-motion functions for a variety of incident seismic wave forms can be computed. As illustrations, the translational and rocking components Δ_y^* and Θ_x^* of the foundation input-motion response vector for the previous embedded foundation due to a vertically incident SH-wave of

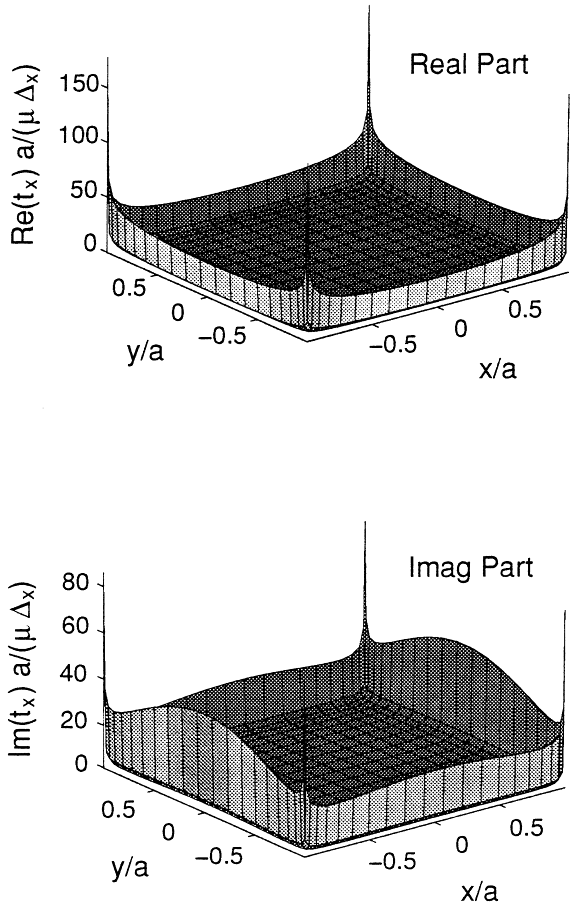

Fig. 9. Contact shear traction due to horizontal translation of rigid surface foundation ($\nu = 1/3$, $\bar{\omega} = 2.5$).

displacement amplitude Δ_{sh} in the y-direction are shown in Figs 14 to 15. From the display, the performance of Mita and Luco (1989) can also be seen to be reasonable.

6. Conclusions

Upon a fundamental examination of the mathematical basis of the analytical theory, a regularized format of the direct boundary integral equation formulation for three-dimensional elastodynamics is established for general applications in solid and geomechanics problems. Founded on the premise of a complete decomposition of the Green's functions into their singular and regular parts, the regularized boundary integral equation formulation is shown to be analytically appealing without demanding mathematical and numerical complexities such as Cauchy principal values. Extended to deal with general seismic soil-structure interaction problems in semi-infinite media, the formulation is implemented computationally together with a consistent singularity treatment

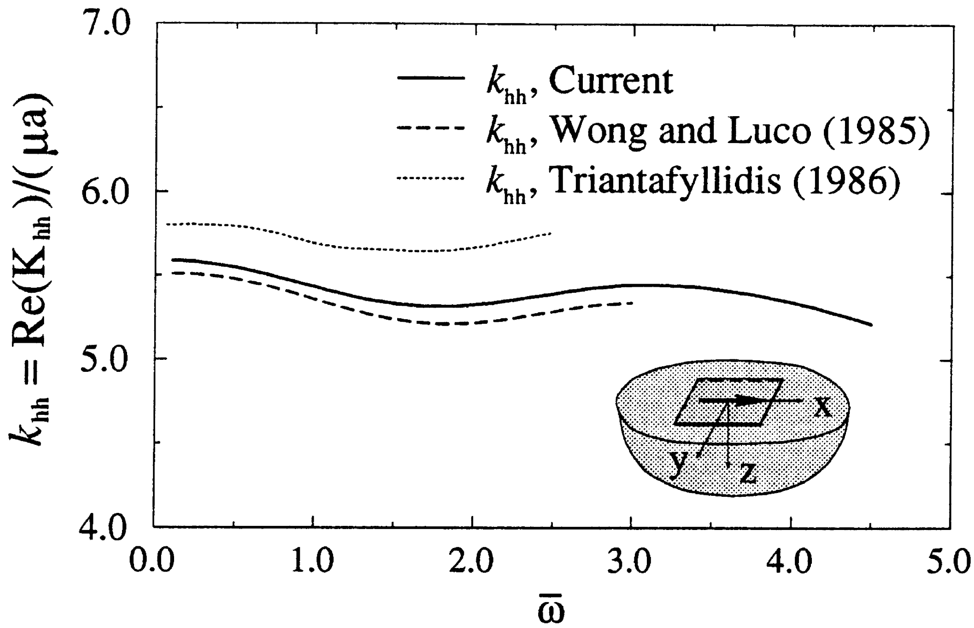

Fig. 10. Stiffness coefficient k_{hh}, square surface foundation ($\nu = 1/3$).

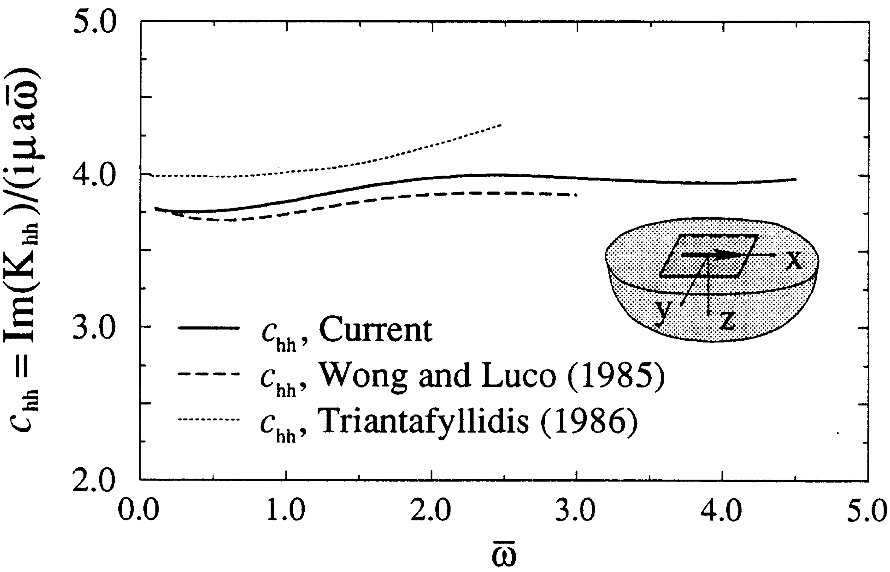

Fig. 11. Damping coefficient c_{hh}, square surface foundation ($\nu = 1/3$).

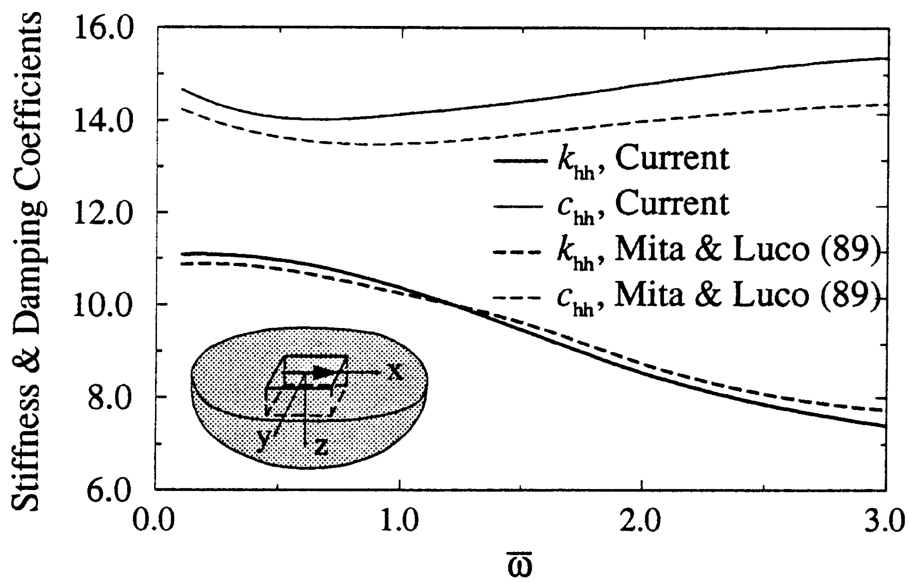

Fig. 12. Dynamic impedance K_{hh}, embedded square foundation ($\nu = 1/3$, $\zeta = 0.002$, $h = a$).

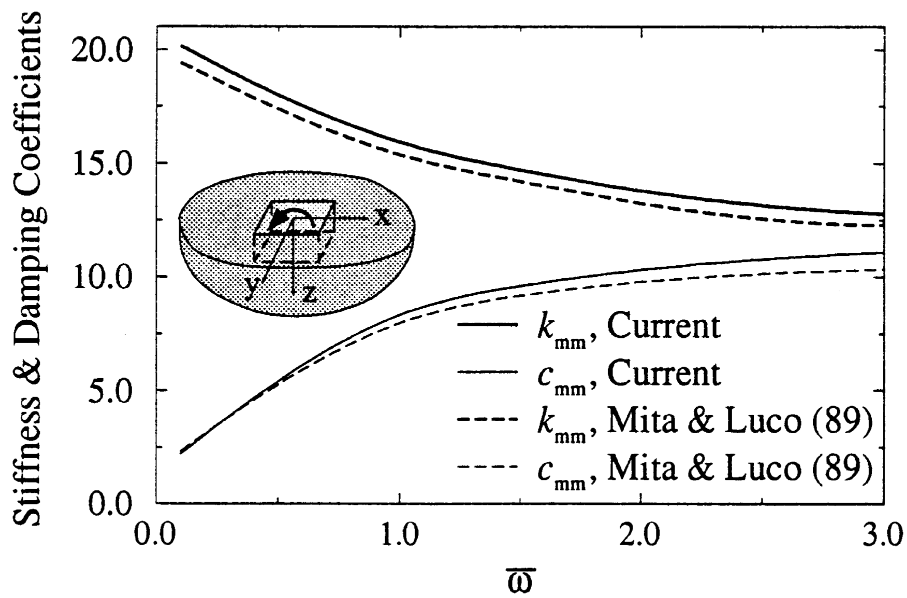

Fig. 13. Dynamic impedance K_{mm}, embedded square foundation ($\nu = 1/3$, $\zeta = 0.002$, $h = a$).

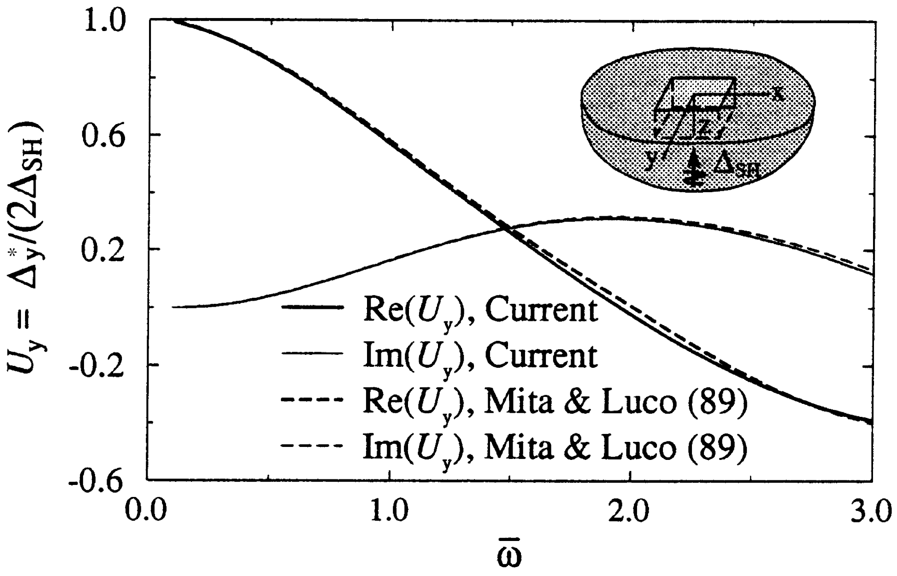

Fig. 14. Foundation input motion U_y due to vertically incident SH-waves ($\nu = 1/3$, $\zeta = 0.002$, $h = a$).

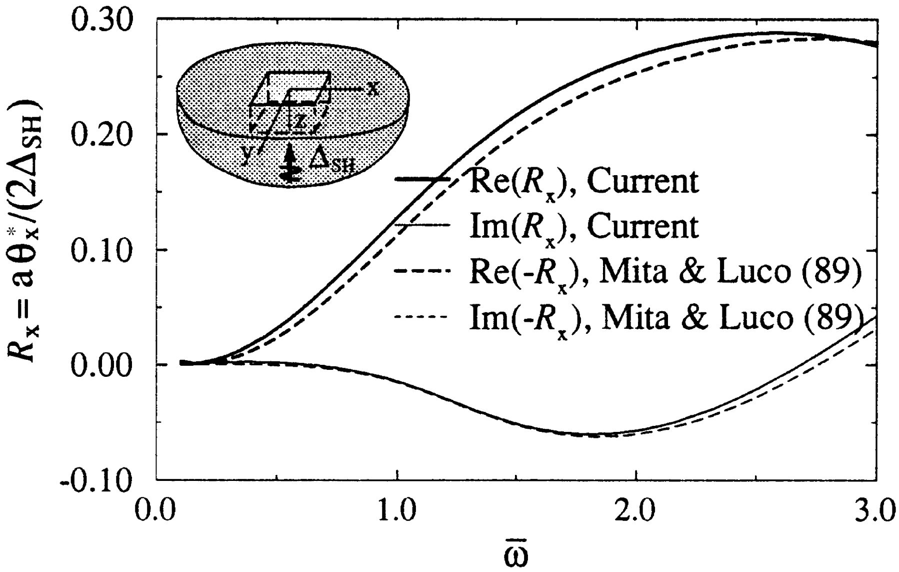

Fig. 15. Foundation input motion R_x due to vertically incident SH-waves ($\nu = 1/3$, $\zeta = 0.002$, $h = a$).

of multi-layered viscoelastic half-space point-load Green's functions as well as a rigorous account of the unbounded contact tractions in the related singular mixed boundary value problems. As illustrations, a set of new benchmark solutions for some basic dynamic soil-structure interaction problems are also included.

Acknowledgements

The support of the National Science Foundation through Grant No. CMS-9320539 under the cognizance of Dr C.J. Astill is gratefully acknowledged.

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